SYLLABUS FOR APPLIED MATHEMATICS’ QUALIFYING EXAM

Applied mathematics focuses on mathematical techniques that yield practical information on the problems of the natural world. The fundamental processes in the natural world are to a large extent described by partial differential equations. This semester focuses on a number of basic partial differential equations and some standard techniques used to analyze these equations. We see how some first order equations can be solved along characteristic curves, how explicit solutions may be found for the wave equation in the entire space, and how the Fourier transform is used to solve linear evolution equations. We learn to analyze the Laplacian operator on a bounded domain. Afterwards we will solve the linear wave equation, the heat equations and Schrödinger equation via various methods. Calculus of variations is a rich source of problems from physical sciences and other branches of mathematics and we learn how this approach leads to differential equations.

Basic References: Applied Mathematics by David Logan and Partial Differential Equations (Graduate Studies in Mathematics) by Lawrence C. Evans.

(1) One-dimensional boundary value problems.
   (a) Eigenvalues and eigenfunctions.
   (b) Sturm-Liouville theory.
   (c) Green’s functions, integral equations, Arzelà-Ascoli theorem.

(2) First order equations, characteristics and shock waves.
   (a) First order equations, the method of characteristics.
   (b) Failure of the characteristic method, weak solutions.
   (c) Shock waves.

(3) Linear elliptic and evolution equation.
   (a) Fourier Transforms.
   (b) The Heat equation, Schrödinger equation and kernels of linear operators.
   (c) Separation of variables.
   (d) The Laplace operator on a bounded region.
   (e) Green’s functions.

(4) Calculus of variations.
   (a) Variational problems, Euler-Lagrange equations.
   (b) The classical harmonic oscillator, the pendulum, minimal surfaces.
   (c) The Dirichlet boundary condition, the Neumann boundary condition.
   (d) Variational problems with constraints.