COLORING, GADGETS AND THE KOCHEN-SPECKER THEOREM

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In this Topology-Quantum Computing talk we explore some mathematical tools for the study of quantum information. In particular, we focus on a graph-theoretic approach to obtaining randomness.

By a $\{0,1\}$-coloring of a graph G we mean an assignment of the values 0 and 1 to the vertices of a graph in such a way that:

1. Two adjacent vertices cannot both be assigned the value 1;
2. Every maximum clique contains a 1.

Graphs which are not $\{0,1\}$-colorable are closely connected to Kochen-Specker sets, a crucial concept in quantum physics. The KS set is usually defined as a set of vectors in $S \subset C^d$ such that there is no function $f : S \mapsto \{0,1\}$ such that:

1. $\sum_{|v\rangle \in O} f(|v\rangle) \leq 1$ for every set $O \subseteq S$ of mutually orthogonal vectors;
2. $\sum_{|v\rangle \in B} f(|v\rangle) = 1$ for every set $B \subseteq S$ of d mutually orthogonal vectors,

but can also be defined in terms of a graph.

We discuss the mathematical properties of non-$\{0,1\}$-colorable graphs and their connection to graph colorings.

AMS Subject Classification: 05C69, 05C05.

References

[1] R. Ramanathan, M. Rosicka, K. Horodecki, S. Pironio, M. Horodecki, P. Horodecki, *Gadget structures in proofs of the Kochen-Specker theorem*, arxiv: 1807.00113