What do lattice paths have to do with matrices, and what is beyond both?

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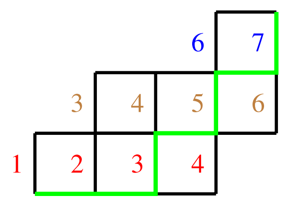
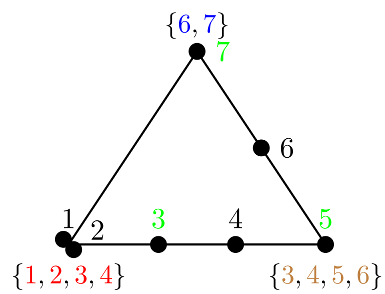


$$N_3 = \{6, \boxed{7}\}$$
$$N_2 = \{3, 4, \boxed{5}, 6\}$$
$$N_1 = \{1, 2, \boxed{3}, 4\}$$

$$\begin{array}{c} \\ N_3 \\ N_2 \\ N_1 \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & * & * \\ 0 & 0 & * & * & * & * & 0 \\ * & * & * & * & 0 & 0 & 0 \end{pmatrix} \end{array}$$



A lattice path is a sequence of east and north steps, each of unit length, that describes a walk in the plane between points with integer coordinates. While such walks are geometric objects, there is a subtler geometry that we can associate with certain sets of lattice paths. Considering such sets of lattice paths will lead us to examine set systems and transversals, their matrix representations, and geometric configurations in which we put points freely in the faces of a simplex (e.g., a triangle or a tetrahedron). Matroid theory treats these and other abstract geometric configurations. We will use concrete examples from lattice paths to explore some basic ideas in matroid theory and some of the many intriguing problems in this field.